APPLICATION OF COHESIVE MODEL IN FRACTURE MECHANICS BY WARP3D

Jarosław Gałkiewicz

Kielce University of Technology, Faculty of Mechatronics and Machine Building
Department of Machinery Design Fundamentals
tel.:+48 41 3424257, fax:+48 41 3442997
e-mail: jgalka@tu.kielce.pl

Abstract

In the paper cohesive model implemented in WARP3D code and an example simulation of crack growth analysis is presented. Cohesive model is an effective tool for a crack growth analysis and it was a main reason to invent it. For the "classica" constitutive equation the crack growth simulation in the finite element method (FEM) is not possible without an additional crack growth criterion.

In the commercial FEM codes cohesive model is not very popular unfortunately. Usually it can be applied as user implemented elements. However there is a code free of charge with high reliability acknowledged in the literature with cohesive elements in standard library. This program is WARP3D and it is dedicated to numerical simulations of three dimensional fracture mechanics problems.

The Dugdale's model, void creation, the cellular model of material, the cohesion-decohesion curve, behaviour of cohesive element, comparison of curve shapes for brittle and ductile fracture, the profile of the cohesive element in WARP3D, specimen geometry, changes of crack shape under increasing load, opening stress distribution in the uncracked ligament are presented in the paper.

Keywords: *finite element method, cohesive model, fracture mechanics,* fracture process,

1. Fundamentals of cohesive model

Papers containing the principles of cohesive model were published in the 60's of 20th century. It is acknowledged that basic rules of cohesive model were postulated by Barenblatt [1] and Dugdale [2], however in fact, the first concept of cohesive zone appeared in the paper by Zheltov and Kristianovich [3] which had been written in Russian and it was not recognized in the international scientific community.

Barenblatt's paper concerning brittle material is concluded with the three hypothesis defining cohesive zone in such a material:

- hypothesis 1: length of a cohesive zone is small with respect to the crack length,
- hypothesis 2: stress distribution within a cohesive zone at the critical moment is always the same and for given material it does not depend on geometry of the specimen,
- hypothesis 3: crack surfaces are tangential at crack tip so the stress singularity disappears at this point.

Barenblatt determined value of the stress intensity factor for a critical moment using equation:

$$K_I = (\sigma_{zw}, a) = -\int_0^d \frac{F_c(\xi)}{\sqrt{\xi}} d\xi . \tag{1}$$

In eq. 1 Barenblatt assumed that distribution of cohesive forces was not known. That was the major disadvantage of Barenblatt model.

This problem resolved Dugdale who analyzed behaviour of thin sheets made of low-carbon steel containing crack. He replaced a complicated, at that time, elastic-plastic problem with an easy

– linear elastic one but with an additional boundary condition. Narrow wedge-shaped plastic zone appearing in front of the crack Dugdale replaced by a one-dimensional region of the same length. In this zone a compressive stress is applied only. The magnitude of the stress is equal to the yield strength (Fig. 1).

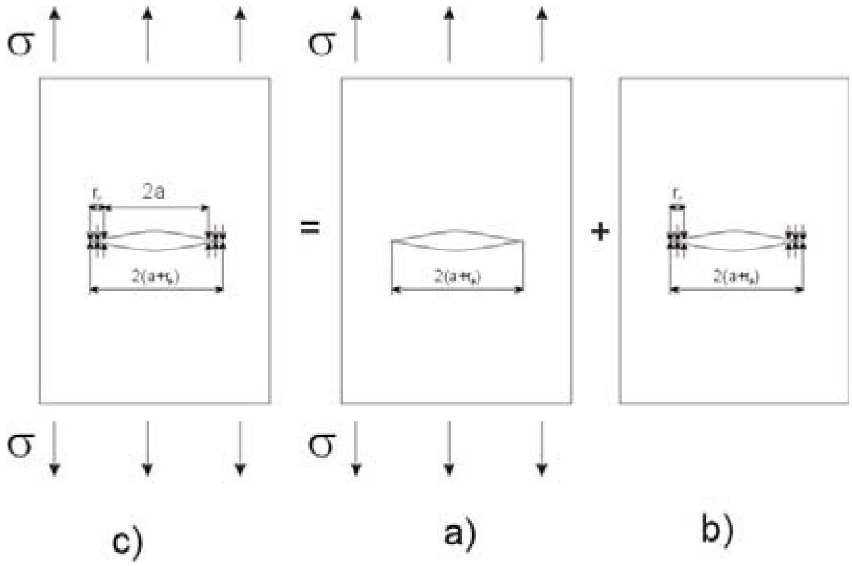

Fig. 1. Dugdale's model

Dugdale's model has some advantages comparing to the Barennblatt's one:
- the length of cohesive zone resulting from equilibrium equations must not to be short,
- unknown stress distribution is replaced by a known compressive stress with constant amplitude which is equal to the yield strength,
- sum of stress intensity factors from the external and internal (compressive) loads is equal to zero in order to cancel the stress singularity. It provides an additional equation to compute the length of plastic zone.

2. Fracture mechanism

Fracture processes take place in a close neighbourhood of a crack tip and they are independent on the size of element. The zone where the fracture processes are active is called a process region. Main feature of the process region is a load level which is so high that it causes a damage of material. That fact influences description of material behaviour. While other part of structure, outside a process zone, can be described by standard constitutive equations, it is impossible for the process zone itself. In this case a continuum mechanics can not be used. Physical processes in process zone differ for various materials and, what is worse, for one material they depend in addition on the environmental issues (temperature, loading rate).

In the case of ductile fracture the most important process is a void nucleation, growth and coalescence. Microseparation takes place near the material's inhomogeneities (Fig. 2).

The most important issue in this process is a distance between dominating nucleation kernels since if crack propagate slowly this dimension controls, in some sense, the size of process region. There are usually several types of particles in structure. The sites where the microseparations nucleate are called dominating nucleation kernels.

Under a considerable plastic flaw the micro-separations nucleate near the stress concentrators. The main reason for this is differences in stiffness of a parent material and a material of inclusion or inhomogeneity. Under an increasing load the stress concentrators may also lead to the microseparation at the matrix-particle interface. Such a defect grows until bridges between particles are destroyed. Alternatively, inclusion can break (especially if they are large and brittle) and void is created due to a matrix plastic flow.

Continuum mechanics does not allow for any discontinuities. Thus, the process zone does not satisfy the fundamental assumption of continuum mechanics. The picture of the fractured surface due to the voids nucleation, growth and coalescence is shown in Figure 2. One may see that in the fracture zone the assumption of the continuous material would be a very strong one. Thus, it is reasonable to replace the small domain in front of the crack by the cellular model of a structure. Such a model is demonstrated in Fig. 3. Each cell should contain one dominating nucleation kernel. Interface between two cells should be placed midway in between two kernels (Fig. 3).

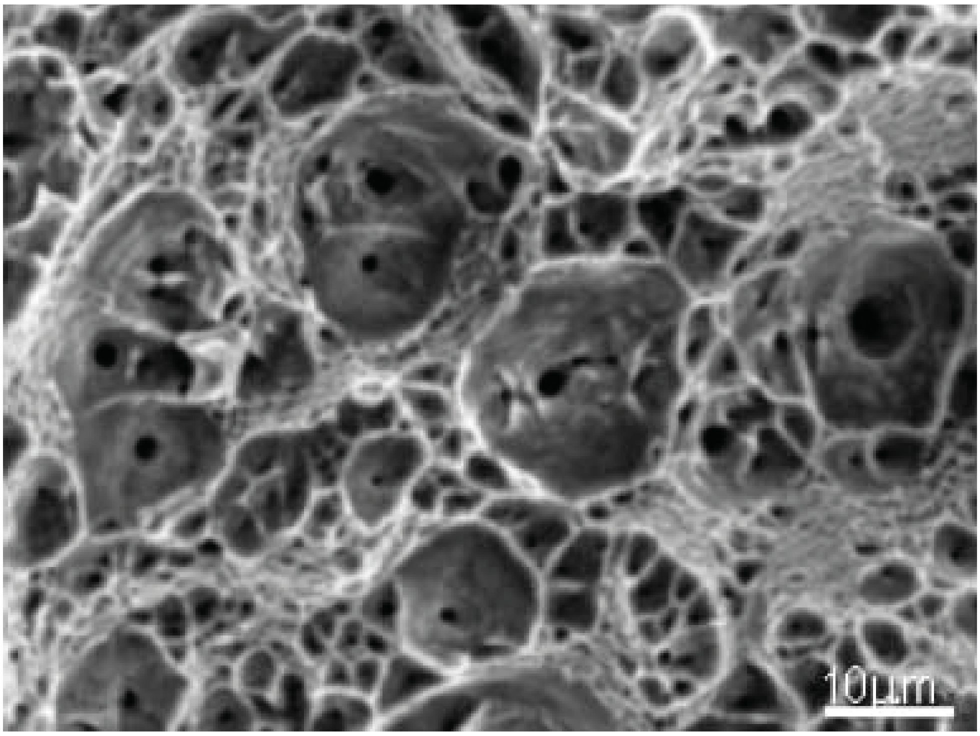

Fig. 2. Void creation

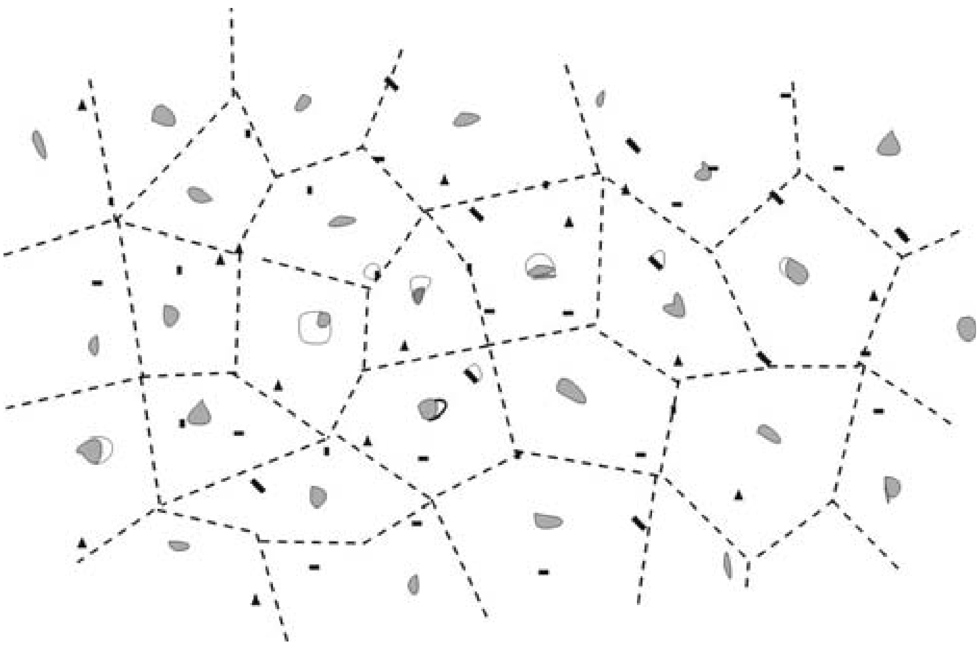

Fig. 3. Cellular model of material

The most important feature of the cell is its behaviour under an applied load. It is modelled by a cohesion-decohesion curve. An example of such a curve for a cube containing the spherical void and loaded by a uniaxial strain is presented in Fig. 4.

The cohesion-decohesion curve consists of two branches: increasing one, which is called - cohesive and decreasing, called-decohesive. Load signifies mean stress acting on a top or bottom surface of the cell. Displacement between these surfaces is denoted by δ. Maximum load $\sigma = \sigma_D$ is reached when the displacement reaches the critical value $\delta = \delta_D$. The curve ends up suddenly as a result of total destruction when $\delta = \delta_{MAX}$. If unloading appears before the critical point displacement decreases as it is shown by the arrow in Fig. 4.

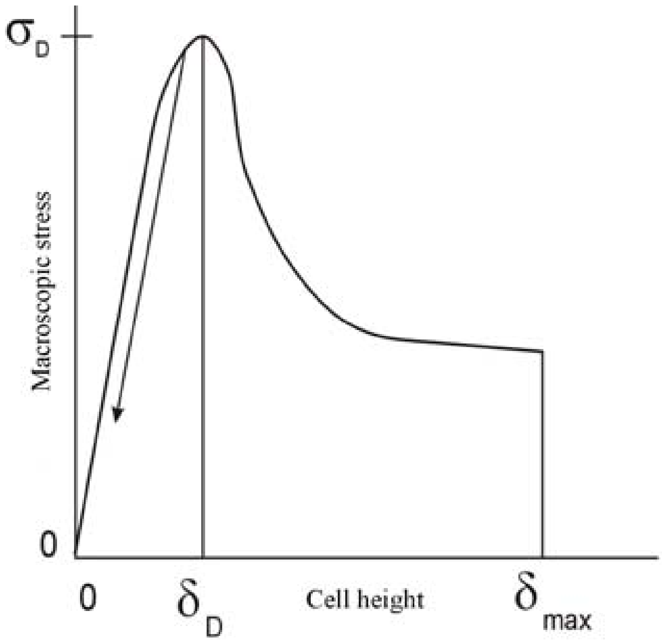

Fig. 4. The cohesion-decohesion curve

3. Assumptions of cohesive model

Basic idea of the cohesive model is a separation of the special zone where the intense degradation of material takes place from a continuum described by the classic constitutive equations. Material in this special zone is described by special equations. This zone was analyzed first in details by Tvergaard i Hutchinson [5] with the help of Gurson's model.

Since it is impossible to derive a precise constitutive equation for a process zone one should postulate a simplified equation. An example representation of such an equation is presented in Fig. 5.

For a full description of material in the cohesive zone some parameters should be defined. Those parameters are divided into two groups. First group contains classic parameters like a yield stress σ_0, Young's modulus E, Poisson's ration ν, Ramberg-Osgood coefficient n. In the other group there are parameters that describe fracture process. There are a maximum stress σ_D, opening displacement at a maximum stress δ_1, opening displacement at the beginning of degradation of the cell, critical displacement δ_c at a cell death, and the most important parameter – the area under a curve which is in fact the work of a separation Γ_0.

The analysis performed by Tvergaard and Hutchinson reveals that some parameters are more important then others. In fact, only two parameters: area under the curve and the maximum stress are significant. Shape of the curve is less important. However, it can provide information on a nature of the fracture process (Fig. 6). Shape presented in Fig. 5. is one of the first which was popular in computations. However, it is not the only one [6, 7, 8].

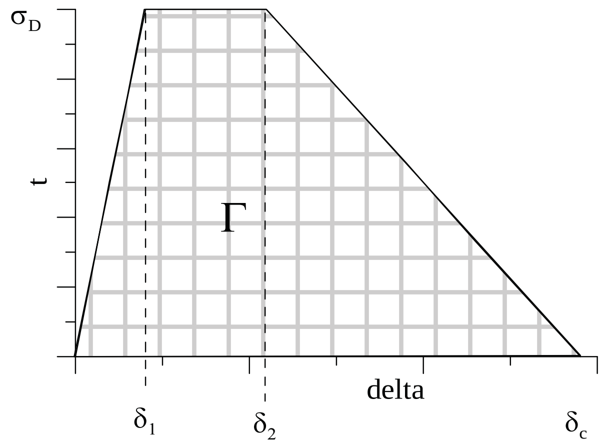

Fig. 5. Behaviour of cohesive element

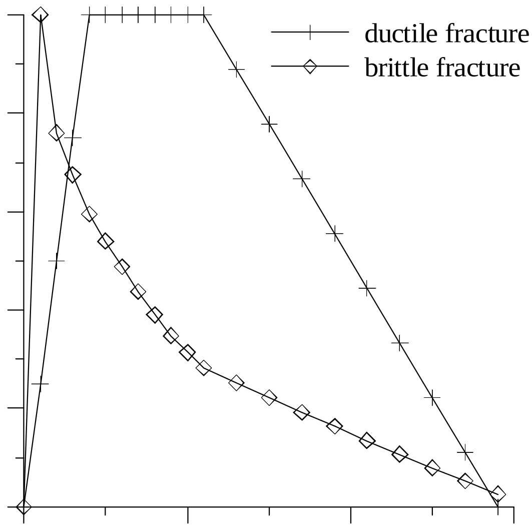

Fig. 6. Comparison of curve shapes for brittle and ductile fracture

4. Determination of cohesive model parameters

The main problem with an application of the cohesive model is determination of the cohesion-decohesion curve parameters. In the literature such a process is divided into two stages usually. In first stage one should determine the value of the parameters according to the some procedure and in the second stage the values obtained should be corrected in order to fit the result of the numerical computations to the experimental results. The area under the cohesion-decohesion curve is identified with the J-integral for a plain strain and a high level of constraint. An easy procedure to determine the critical stress level was proposed by Brocks et al [9]. According to them the critical stress is equal to the stress level in the centre of a notched specimen in the critical section at the moment of rupture during the tensile test.

5. Application of cohesive model in WARP3D

Cohesive model opens new fields in analysis of fracture process. It allows an analysis of a crack growth in the 2D and more complex, 3D structures taking into account an influence of the in-plane and out-of-plane constraint in a mixed mode of loading. Theoretically, the cohesive model is able to simulate the crack growth for a crack length equal to zero. Thanks to its capabilities the cohesive model is more and more often utilized in the numerous numerical computations. Unfortunately, in the most popular commercial codes one can apply it by the user created libraries only. It decreases a reliability of such computations.

Among the rare programs, that contain positively verified cohesive elements in their standard versions, is WARP3D. The program has been developed at the University of Illinois. Its reliability has been confirmed by numerous research results [10, 11, 12]. Additional advantage of the program is its freeware distribution, which can be download from ftp server: **cee-ux49.cee.uiuc.edu** (user: **ftp**, password: **ftp**) and detailed user manuals with the examples.

In the code the following relation to describe the cohesive element is used (Fig. 7):

$$t = \frac{\partial\phi}{\partial\delta} = e\sigma_c \frac{\delta}{\delta_c} \exp\left(-\frac{\delta}{\delta_c}\right), \tag{2}$$

where *t* is the opening stress, δ is the effective displacement ($\delta = \sqrt{\beta^2\delta_s^2 + \delta_n^2}$), δ_n is the normal displacement, δ_s is the tangent displacement, β is the coefficient for mixed mode computations, e = exp(1).
The relation under curve can be obtained by integration of eq. 2:

$$\Gamma_c = e\sigma_c\delta_c\,. \tag{3}$$

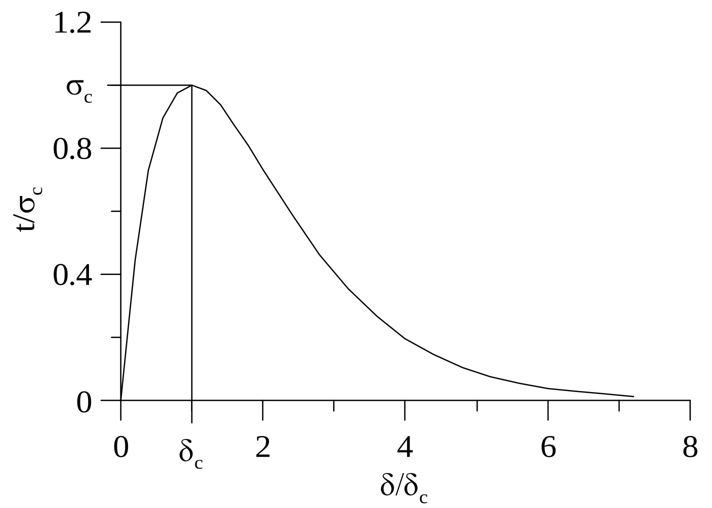

Fig. 7. Profile of the cohesive element in WARP3D

Cohesive element has additional requirements with respect to the standard 3D finite elements. The height of the element before computations started is equal to zero. Nodes in the element should be listed in right turn. The first set is defined at the lower layer and then at the upper one. The order of nodes should be defined in such a way that the normal to the plane created by nodes is in the direction towards the interior of the element.

Results of computations can be stored in different type of files. The most suitable are the PATRAN formatted files for selected load steps since there are many tools available for further processing of the results.

6. Example problem

As an example the three point bending specimen is presented. The model of specimen is shown in Fig. 8. Width of the specimen is 24 mm, the span 96 mm and the initial crack length is 11.5 mm. The specimen is loaded by the displacement as it is presented in Fig. . The maximum displacement is 1.4 mm.

Specimen is made of 40 H steel, which was tempered at 850°C and quenched at 680°C. The properties of the material are presented in Tab. 1.

Tab. 1. Properties of the 40H steel

Material	Heat treatment		Mechanical properties			Ramberg-Osgood parameters		Fracture toughness
	temper	quench	R_e (MPa)	R_m (MPa)	hardness (HRC)	α	N	J_{IC} [kN/m]
40H	850°C	680°C	700	820	29.5	1	14.5	246

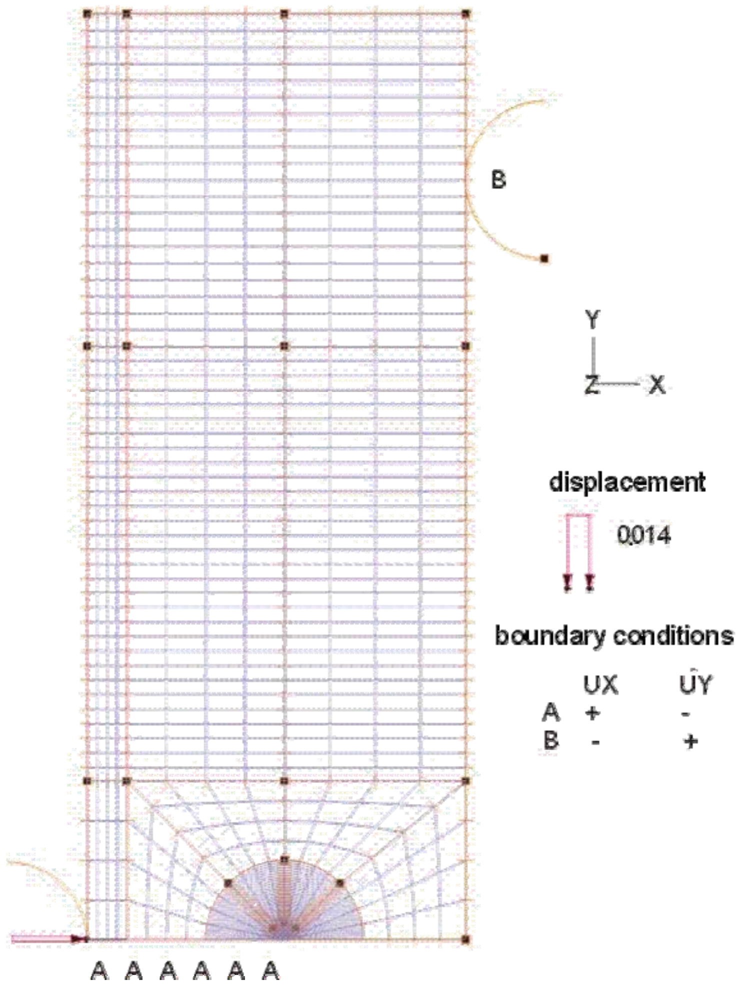

Fig. 8. Specimen geometry

Cohesive element were implemented along the line with the boundary condition ,,A" (Fig. 8). Since the computations are performed to illustrate the method only it was assumed that the maximum stress in the cohesive zone can reach the double value of the yield stress that is 1400 MPa. For this assumption the critical displacement is equal 0.03232 mm.

7. Results

Figure 9 presents displacement for the nodes along the line y = 0 (symmetry line). The x-coordinate takes into account displacement of specimen under an external load. It is clear that the crack tip opening angle changes initially very fast. However, at the end of computations the lines representing the crack edge are parallel – this justify the application of a crack opening angle as a good parameter used often for the crack growth problems.

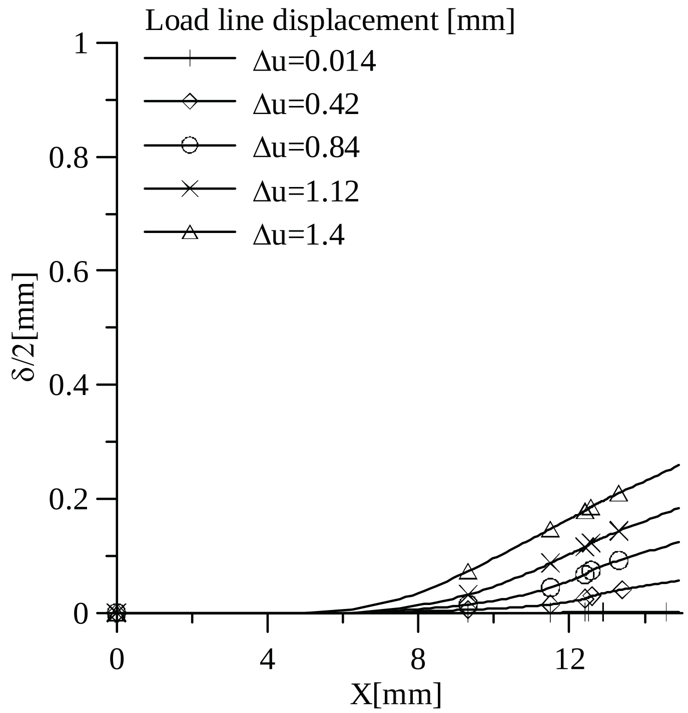

Fig. 9. Changes of crack shape under increasing load

In Fig. 10 the opening stress is presented.

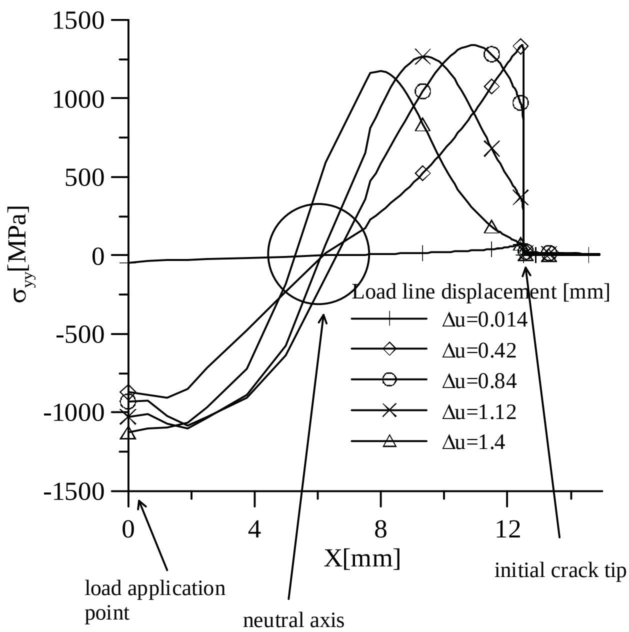

Fig. 10. Opening stress distribution in the uncracked ligament

The stress level in the presented case increases up to the moment of the onset of a crack growth. Then the maximum opening stress decreases slightly. Such behaviour of the stress field does not come along with the predictions if one does not take into account dynamic effects. This means that the model requires, probably, some modifications. In the case of a bended specimen the level of the opening stress in front of the crack increases for longer cracks and greater deflection.

8. Conclusions

Presented method is not a new one. However, the analysis presented aimed at the popularization of the cohesive model and the WARP3D code which are the best for the numerical computation in the field of fracture mechanics and are not known and not used in Poland according to the author's knowledge.

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